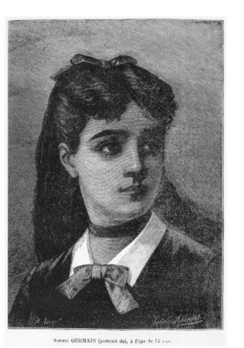
Marie-Sophie Germain (mathematician and physicist)

April 1, 1776 – June 27, 1831

Marie- Sophie Germain was a French mathematician who was born during a time when women were not allowed to study mathematics and the sciences. She grew up during the French Revolution, and her parents kept her indoors for safety. She spent her time in her father's library which had a vast collection of books. She taught herself mathematics and the other sciences. She also learned Latin so she could read books written in Latin. Sophie was able to penetrate the exclusive circle of prominent mathematicians during her time by assuming the name of a male student at the Ecole Polytechnique. She used a man's name as her pseudonym, in order to be taken seriously by the male mathematicians in the 18^{th} century; just like female gamers who pretend to be male in the twenty-first century.

Her science journey began when Marie-Sophie Germain was born on April 1, 1776, in Paris, France. She was the middle child of a prosperous merchant, goldsmith, and jeweler who later became a silk merchant, Ambrose-Francois Germain d'Orsanville and his wife Marie-Madeleine Gruguelu. Her paternal grandfather Thomas Germain was a sculptor and architect, while her maternal grandfather Jean Gruguelu was a goldsmith and a friend of philosophers and political economists. Ambroise-François was elected a deputy to the National Assembly in 1789 and later became a director of the Bank of France. Marie-Sophie had two sisters, Marie-Madeleine who was five years older and Angelique-Ambrose who was two years younger. Because both her mother and sister had Marie in their names, Sophie decided to drop the Marie and just go by the name Sophie. She was born ten months after Louis XVI was crowned King of France and his wife Marie Antoinette became Queen. When Louis the XVI became King, he inherited a kingdom that was almost bankrupt, and later a harsh winter resulted in failed crops which contributed to high prices of bread and other foods. The French economy deteriorated with



rising unemployment. The French people blamed Marie Antoinette, who was a foreigner from Austria. King Louis XVI tried to reform the monarchy by abolishing serfdom and removing the taille, a tax levied on common people. But the discontent was too much, and a rebellion exploded in Paris and throughout France. On July 14, 1789, when Sophie was 13 years old, mobs of angry citizens stormed the Bastille, a prison located near Sophie's house on Rue St. Denis. The French revolution would see more bloodshed; such events accompanied Sophie throughout her childhood. Her parents tried to shield her and her sisters from the Revolution by keeping them at home. Sophie used this time confined at home to read all the books in her father's library. Her father's collection of books contained volumes on different subjects, but it was mathematics that she was most interested in studying. Her parents did not want her to study mathematics as it was a subject that is not appropriate for girls and women. She would sneak out of bed to study math while the others slept. The parents worried that this would bring heartbreak and scorn to their daughter. So, they took away the candles in her room, stopped lighting the fireplace in the room and removed all her warm clothes. But that didn't stop her. One morning, they found her asleep at her desk bundled in her blankets next to a pot of ink that had frozen solid. Apparently, nothing could stop her from studying mathematics, so her parents relented. It was impossible to become a female mathematician anyway!

At her father's library she read a book called *Complete Course in Mathematics* by Etienne Bezout. Then she read *the History of Mathematics* by Jean-Etienne Montucla. Sophie was inspired by the story about Archimedes, one of the greatest mathematicians and scientists of the ancient world. The story of his death at the hands of a Roman soldier – in which he could not be bothered to stop his drawing of geometric figures on the sand floor – impressed the young girl. She then decided that mathematics would be her field of study and that she would pursue it with a single-minded effort just like Archimedes. Sophie knew that to pursue a career in mathematics, she had to familiarize herself with the work of Isaac Newton, Leonhard Euler, and others who had made significant advances in the field. She taught herself Latin so she could study the works of Isaac Newton.

ECOLE POLYTECHNIQUE

In 1794, the Ecole Polytechnique opened in Paris. The school offered an alternative to the

institutions used to prepare young men for military service. Top scientists in France were recruited to teach at this school. Sophie of course could not attend this school because she was a woman. Shortly after the school opened, Sophie met a young man, Antoine-August LeBlanc, who was a student at the Ecole Polytechnique. He shared his lecture notes with Sophie. But then he was called to military service and died at the young age of 22 years old. LeBlanc did not tell the school that he was leaving for military service and so Sophie continued to receive the lecture notes printed for LeBlanc. This presented a great opportunity for Sophie! In 1795 or 1796, she began submitting written works under the name of M. LeBlanc. Her identity was discovered

inadvertently when a distinguished mathematics professor named Joseph-Louis Lagrange read the paper Sophie submitted as M. LeBlanc. Lagrange was one of the greatest mathematicians of the 18th century. Lagrange was so impressed with the originality of the paper that he decided to track down the student. He went to visit him at his house on rue St. Denis and was shocked when he found out that M. LeBlanc was actually a woman. He offered to mentor her, and Sophie then became a local celebrity in Paris and in the field of mathematics. Female mathematicians were almost unheard-of in France in the 18th century! But Sophie was not interested in being known as a female mathematician. She wanted to be known as a good mathematician!

Mathematicians and scientists from all over Paris came to see her. Some gave encouragement and help like the mathematician "Citizen" Cousin who had written a textbook *Lessons on Differential and Integral Calculus* and who offered to help with all the resources he possessed to help her in the practice and profession of science. Others like the astronomer Jospeh-Jerome Lefrancais de Lalande offended her by telling her to read his book *Astronomie des dames* (*Astronomy for Women*) because that's the only way women could understand a complex scientific topic like astronomy. Lalande even went to visit Sophie at her house and when she informed him that she had already read the more technical *Exposition du système du monde (The System of the World),* Lalande told her he believed she would not understand that book unless she read *Astronomy for Women* first. Her dislike of Lalande became well known in Paris!

In 1801, Carl Friedrich Gauss published a book titled *Disquisitiones Arithmeticae* (Arithmetical Investigations). Gauss is one of the greatest mathematicians in history! His book transformed the field of number theory. Gauss' book inspired Germain to work on problems in number theory. In 1804, she wrote to Gauss under her pseudonym M. LeBlanc. Her initial letter was to compliment Gauss but soon they corresponded for months and years. Gauss was also impressed with the knowledge and ability of this French mathematician. He even mentioned his impression of his friend to the astronomer Heinrich Olbers. Her identity would not have been discovered if not for a series of events that happened in 1806 when Napoleon Bonaparte was at war with Prussia. Sophie found out that Napoleon's forces were in Braunschweig, the hometown of Carl Gauss. Fearing that Gauss would suffer the same fate as Archimedes, she contacted her friend Joseph-Marie Pernety, who was a general in Napoleon's army. She asked him to guarantee Carl Gauss' safety. When Gauss heard that a certain Sophie Germain and her friend Joseph -Marie Pernety were protecting him, he replied that he doesn't know them. So, Sophie was forced to tell him that she was actually M. LeBlanc. Gauss was surprised that his friend was a woman, but he nevertheless encouraged her to continue her

studies. They continued their correspondence until 1808 when Gauss received a prestigious university appointment in Gottingen, Germany. Then his interests shifted from number theory to a wide variety of academic disciplines such as magnetism and astronomy. His last letter to Sophie was dated January 19, 1808. Sophie continued studying with French mathematicians Joseph -Louis Lagrange and Adrien-Marie Legendre.

ELASTICITY

When she was 32 years old, Sophie witnessed a scientist sprinkle sand onto a glass plate and then rubbed the violin bow against the plate's edge producing vibrations that shook the glass until it rang out with a sound. She was surprised to see the sand dance across the plate! Then it formed circles and diamonds and figure eights! The higher the note the more quickly the vibrations shook the plate and formed more intricate sand patterns! Suddenly Sophie realized that everything she did, like knocking on the door or walking on the cobblestones with her boots sent vibrations to nearby objects just as waves flowed through water. Everyone in Paris was fascinated with this new phenomenon! This phenomenon can affect buildings and bridges! How much vibration was too much? At what point would an object break? The demonstration of vibrating plates was done by a German physicist Ernst F.F. Chladni. When he arrived in Paris in 1808, he was already a familiar figure among European intellectual circles. His first book in 1787 was about the attempts he had made to understand how sound moves through space and to measure the speed of sound through various gases. He was also considered to be the founder of meteoritics or the study of meteorites. Chladni's vibrating plate also caught the attention of Napoleon Bonaparte. The French emperor suggested that the First Class of the Institute, France's most prestigious scientific society offer a prize to anyone who could develop a mathematical theory explaining the vibration of elastic surfaces. France's scientific societies had a long tradition of sponsoring such competitions. Previous winners include Leonhard Euler, the great Swiss mathematician. In 1793, amidst the French Revolution, the Paris Academy of Sciences was abolished. Two years afterwards, in 1795, the National Institute of Sciences and Art was established. It was divided into several "classes", the first being for physical science and mathematics. Thus the First Class of the Institute became the most prominent scientific organization in France. In April, 1809, the First Class of the Institute offered a medal worth 300 francs to anyone who could find a mathematical formula that would predict patterns of vibration.

This new competition called *prix extraordinaire* piqued the interest of Sophie Germain. In 1779, Leonhard Euler had derived a differential equation to describe the vibrations of an elastic beam. Sophie thought she could expand on Euler's work. She worked on the solution to the problem for two and a half years. Two other mathematicians, Lagrange and Pierre Simon de Laplace got interested in the competition because they thought that this was the kind of problem their protégé Simeon -Denis Poisson was well equipped to solve. Winning the *prix extraordinaire* would greatly advance Poisson's career.

On September 11, 1811, a month before the contest closed, Sophie submitted her entry to the First Class of the Institute. A month later, Sophie was the only entrant in the contest. The Judging Committee which included Adrien-Marie Legendre checked her paper line by line, equation by equation. It took months for the Committee to evaluate the paper Sophie submitted. Their conclusion was that Sophie had not sufficiently explained the phenomenon of vibrating plates. However, the judges did not argue with her hypothesis, that the elastic force of the plate is proportional to its curvature. But they pointed out many mistakes and exposed her deficiencies in her mathematics studies. She had not had formal training in mathematics, so her education was haphazard. In moving from hypothesis to differential equation, Sophie did not successfully use Lagrange's calculus of variations. In his formal evaluation, Lagrange presented the equation that she would have derived had she correctly employed rigorous mathematics to her hypothesis. Lagrange figured out this equation based on Sophie's highly original paper. Because the problem of vibrating plates remained unsolved, the First Class of the Institute announced a continuation of the previous contest. The deadline was extended by two years. Although Lagrange was able to derive a suitable equation, he could not find an explanation for why curvature should affect the vibrations. In her second competition entry, Sophie attacked the problem from a different angle. After two years, Sophie was still the only contestant. The committee reviewed her new entry and concluded that her equation was truly that of the vibrating surface. But she still had not adequately explained Chladni's plates. Legendre told her that he believed she did not have a very clear idea of the operations on double integrals in the calculus of variations. She was awarded an honorable mention. The First Class of the Institute opened the competition for a third and final time. The deadline was October 1, 1815.

In August 2014, Dennis Poisson presented a paper to the Institute which included the equation Lagrange had derived from Sophie's paper; but he did not mention Sophie's work. Poisson presented the research as though it was his own. Poisson had been a contestant during the first competition but became ineligible when he was admitted to the First Class of the Institute.

Since the competition was still open, Sophie attacked the problem anew, this time changing her hypothesis. Instead of trying to specifically connect the elastic force to the curvature of the plate, she simplified her work by saying at the outset that the elastic force was connected to the ways in which the plate was deformed. She also added a section addressing plates that were initially curved. For the third time she was the only entry. After evaluating the paper, the Committee concluded that Sophie had won the *prix extraordinaire*! For the first time, a woman received the highest honor in the history of the First Class of the Institute! The judges said that her work did not correctly explain all vibrating surfaces, but it did however explain the vibrating surfaces of Chladni's plate.

The theory of elasticity, the modern term given to the work Sophie was doing in the 19^{th} century, continued to evolve and was used in a variety of applications. One was the construction of buildings. To create tall structures like the Eiffel Tower, architects and engineers must understand how the wind and the motions of the earth will cause the building to move.

The next *prix extraordinaire* which was set in 1816 took Sophie back to her first love, number theory. Now forty years old, she entered this contest, which was about one of the most famous unsolved problems in mathematics; Fermat's Last theorem. Unlike the problem of Chladni's plate which dealt with applied mathematics, this problem was purely theoretical. The problem first emerged in the seventeenth century as Pierre de Fermat's last "theorem". He stated that there are no natural numbers (1, 2, 3,…) x, y, and z such that $x^n + y^n = z^n$, where n is a natural number greater than 2. Fermat posed the problem and said he has a proof but he never wrote up a proof. Sophie tried to reconnect with Carl Gauss as she worked on proving Fermat's Last Theorem. But Gauss never responded. By factorization, mathematicians had proved that Fermat's Last Theorem was true for all even numbers n and in fact for all composite numbers. What remained was to prove that it works for prime numbers n. So Sophie worked on the prime numbers, and made the best progress up to that time by focusing on a certain class of primes. She never published her work but appeared in the memoirs of Legendre and other papers written later. These prime numbers that she studied have come to be called Sophie Germain primes after her. "Sophie Germain's Theorem" has been presented by Legendre to the French Academy of Sciences in 1823. It provided a roadmap to prove Fermat's Last Theorem for certain groups of numbers. She used her new method to prove Fermat's Last Theorem for every prime number less than 100. In 1994, a mathematician at Princeton University proved Fermat's Last Theorem to be true for all numbers, using mathematical tools of the 20^{th} century which would be unimaginable in Sophie's lifetime.

Even after winning the *prix extraordinaire*, Sophie still had a hard time gaining the respect of her fellow mathematicians and scientists. She was not allowed to join the Paris Academy of Sciences (which had been reestablished) or attend the lectures of the First Class of the Institute. When she heard that Poisson did not give her any credit for her work on elasticity, she decided to publish it herself in 1821 with help from Jean-Baptiste-Joseph Fourier. When Fourier became the permanent secretary of the Paris Academy of Sciences in 1822, one of his first official acts was to tell Sophie that she could attend the scientific lectures of the Institute of France not as a wife or guest of a member but as a scientist in her own right. This was the first time a woman has been given this honor!

Sophie continued her research in mathematics, helping younger mathematicians in later years. In 1825, she met a young mathematician from Pisa, Italy at a party hosted by the astronomer Francois-Jean Arago. They became friends even though he was 26 years younger than her. Much of what we know about Sophie Germain came from her biography written by this mathematician Guglielmo Libri. Over the next four years they corresponded regularly.

In 1829, Sophie was diagnosed with breast cancer. She died on June 27, 1831 at the age of fifty-five. Before she died, she published her last paper on curvature in 1830. Libri wrote an obituary for his friend. Six years after her death, when the University of Gottingen was considering possible recipients for honorary degrees, Carl Friedrich Gauss suggested that Sophie Germain receive a special honor. It was only then that he learned of her death. When the Eiffel Tower was built and the names of 72 scientists and engineers were inscribed on the structure, no one mentioned the name of Sophie Germain. Was she excluded from this list because she was a woman?

HER LEGACY

Sophie Germain made many contributions to the field of mathematics and mathematical physics. Her work was fundamental in the development of the general theory of elasticity. The *Germain curvature* or mean curvature of a surface is an extrinsic measure of curvature that comes from differential geometry and that locally describes the curvature of a surface embedded in a larger space such as Euclidean space. The concept was used by Sophie Germain in her work on elasticity. She will also be forever known for the Sophie Germain prime. A Sophie Germain prime is a prime number that, when multiplied by 2 and then increased by 1, yields another prime number. The first few Sophie Germain primes are 2, 3, 5, 11, 23, 29, 41, 53, 83, 89, 113, and 131. In 2003, the Sophie Germain Prize which is called *Prix Sophie Germain* in French, was established by the Academy of Sciences in Paris to honor a French mathematician for research in

the foundations of mathematics. The Prize comes with a monetary award of €8,000. At the centennial celebration of her life, a street and girls' school were named after her. The Lycée Sophie Germain is a public, general and technology lycée in the 4th arrondissement of Paris. Rue Sophie Germain runs between Rue Halle and Avenue du General Leclerc, opposite the Mouton Duvernet Metro station. A commemorative plaque was placed in the house where she died.

She was referenced and quoted in David Auburn's 2001 play *Proof.* The protagonist was a young, struggling, female mathematician who was inspired by the works of Sophie Germain. A movie with the same name was shown in the theatres in 2005. In the fictional work "The Last Theorem", Sophie was credited with inspiring the main character to solve Fermat's Last Theorem.

In January 2020, Satellogic, a high -resolution Earth Observation imaging and analytics company launched a ÑuSat type micro-satellite named in honor of Sophie Germain.. A musical about Sophie Germain's life entitled *The Limit* was shown in London in 2019.

HER SCIENCE JOURNEY

1776 - Marie- Sophie Germain was born in Paris, France

1789 - French Revolution began, Sophie confined at home started reading the books in her father's library and discovered the wonderful world of mathematics

1794 - Ecole Polytechnique opened

Met Antoine-August LeBlanc

1795 - Began submitting work under the pseudonym Antoine-August LeBlanc and was discovered by Joseph -Louis Lagrange

1801 - Read *Disquisitiones Arithmeticae* by Carl Friedrich Gauss

1804 - Wrote to Gauss using the pseudonym Antoine-August LeBlanc

1806 - Asked her friend General Pernety to ensure the safety of Carl Friedrich Gauss during the invasion of, Braunschweig, Germany

1809 - Started working on the first *prix extraordinaire*

1811 - First attempt at *prix extraordinaire*

1813 - Second attempt at *prix extraordinaire*

1815 - Third attempt at *prix extraordinaire*

1816 - Won the *prix extraordinaire,* continued study of number theory and Fermat's Last Theorem

1821 - Published work on theory of elasticity

1822 - Allowed to attend lectures at the First Class of the Institute

1830 - Final paper on analysis of curvature

1831 - Died of breast cancer at 55 years old in Paris

1876 - Lycée Sophie Germain and Rue Sophie Germain were named after her. A plaque commemorating her death was placed in front of the house where she died.

2001 - The play *Proof* about a young, female mathematician was inspired by Sophie Germain

2003 - The Sophie Germain Prize was established

2005 - The movie Proof was shown in the theaters

2019 - A musical about Sophie Germain's life entitled *The Limit* was shown in London in 2019

2020 - Satellogic, a high -resolution Earth Observation imaging and analytics company launched a ÑuSat type micro-satellite named in honor of Sophie Germain

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